Homework 4

(Due Thursday, Feb. 21)

Additional Problems 1-3 Apply Euler's method to approximate the solutions of the following differential equations at $t = 1$, starting from the given initial condition. First use one step with $\Delta t = 1$, and then use two steps with $\Delta t = 0.5$. Compare to the exact value of the solution (given below) at $t = 1$.

(1) $\frac{dx}{dt} = t, x(0) = 1$ (actual solution: $x(t) = 1 + \frac{t^2}{2}$)
(2) $\frac{dy}{dt} = 2y, y(0) = 3$ (actual solution: $y(t) = 3e^{2t}$)
(3) $\frac{dz}{dt} = z^2, z(0) = 1$ (actual solution: $z(t) = \frac{1}{1-t}$)

Chapter 5.1: 23, 25, 27-28 (which pair with 5.2, 25-26, and 5.3, 21-22), 48

Chapter 5.2: 1-4, 15-18 (do 1 and 15 together, 2 and 16, etc.), 25-26 (see above)

Chapter 5.3: 21-22 (see above – all you need to do here is determine the stability of all fixed points), 17-20

Additional Problems 4-5 Active transport of ions can be modeled as a constant stream of ions flowing into a cell at rate k. Ions leave the intracellular medium by uptake, buffering, or passive diffusion at a rate proportional to the intracellular concentration C.

(4) Using the modified concentration ODE

$$\frac{dC}{dt} = F(C) = k - \lambda C$$

graph $F(C)$ against C and draw a phase line. What is the equilibrium concentration in terms of k and λ? Show that it is stable (as long as $\lambda > 0$).

(5) Now consider the case in which the rate of active transport increases as a "sigmoid" function of intracellular ion concentration. We will use the sigmoid function

$$s(C) = \frac{\tan^{-1}(8C - 4)}{\pi} + 1/2$$

which rises quickly from nearly zero to nearly one around $C = \frac{1}{2}$. Use a graphing program (see below) to sketch the graph of the ion flux $s(C)$ as a function of concentration C. Next, sketch the graph of

$$\frac{dC}{dt} = F_2(C) = s(C) - C$$

and draw a phase line. Thoroughly describe what is going on here.

I suggest Wolfram Alpha as a graphing program. Go to http://wolframalpha.com and type:

plot arctan(8x-4)/pi + 1/2

Third Edition check:

- 5.1 #48 begins, "What is the relationship between r and λ?"
- 5.2 #25-26 refer back to 5.1, #27-28.
- 5.3 #20 begins, "Consider the equation $\frac{dx}{dt} = ax + x^3$..."

If anything on this assignment looks like it might be a typo, do not hesitate to email me about it.