Homework 7

(Due Thursday, April 4th)

Part 1

In many biological system, e.g. the endocrine system, processes are regulated by delayed feedback. Delayed feedback is great for creating periodic behavior.

(1) Consider two strategies for the regulation of hormone concentration:

$$H_{t+1} = H_t - \frac{1}{2}(H_t - 1)$$
$$H_{t+1} = H_t - \frac{1}{2}(H_{t-1} - 1)$$

The first model portrays immediate negative feedback: some production/uptake process responds immediately to the hormone level and corrects it towards 1. The second model portrays delayed negative feedback: some production/uptake process responds to the hormone level, but only corrects it after one time step.

(a) Graph the solution to the first model up to $t = 6$, with initial condition $H_0 = 3$.
(b) Graph the solution to the second model up to $t = 6$, with initial conditions $H_0 = 3$ and $H_1 = 3$. (The first thing you will calculate is H_2.)
(c) Explain in words why the second model produces qualitatively different solutions.

(2) Consider two strategies for the regulation of hormone concentration:

$$J_{t+1} = J_t - \frac{3}{2}(J_t - 2)$$
$$J_{t+1} = J_t - \frac{3}{2}(J_{t-1} - 2)$$

(a) Graph the solution to the first model up to $t = 6$, with initial condition $J_0 = 3$.
(b) Graph the solution to the second model up to $t = 6$, with initial conditions $J_0 = 3$ and $J_1 = 3$. (The first thing you will calculate is J_2.)
(c) How might the endocrine system prevent this type of behavior? Assume that the delay is unavoidable.

Part 2

Many papers on neural network modeling use a very simplified model of the neuron: the Leaky Integrate and Fire (LIF) model. The LIF model follows the differential equation

$$\frac{dV}{dt} = F(V) = C - V$$

...where C is the amount of synaptic current entering the neuron. *When* V *reaches 1, a "spike" occurs, and* V *instantly is reset to* 0. Thus, V can never be greater than 1.

(1) Let $C = \frac{1}{2}$. Where is the fixed point for V? Is it stable? Assuming $V(0) = 0$, what happens?
(2) Let $C = 1$. Where is the fixed point for V? Is it stable? Assuming $V(0) = 0$, what happens?
(3) Let $C = \frac{3}{2}$. Where is the fixed point for V? Is it stable? Assuming $V(0) = 0$, what happens?
(4) Let $C = \frac{3}{2}$ again. Solve this equation for $V(0) = 0$. At what time t does $V(t) = 1$?

If spikes are occurring regularly, the time between spikes is called the "period," which we will denote as T. The "spike rate" is defined as $\nu = \frac{1}{T}$.

(5) Let $C = 2$. Solve this equation for $V(0) = 0$. What is the spike rate?
(6) For what values of C does regular spiking occur? For a fixed value C in this range, what is the spike rate in terms of C?

Extra Credit:

Consider the situation in which $C(t)$ varies with time. Specifically, let $C(t)$ alternate periodically: it is 0 for an amount of time τ_1, and then some positive value $C_{max} > 1$ for an amount of time τ_2, then 0 again... The intervals during which $C(t) = C_{max}$ are "pulses." Under what circumstances does a spike occur during every pulse? Is it possible for spikes to occur only on every second or every third pulse? Give me as much information as possible. Feel free to assign values to the parameters to gain intuition for the problem.

Part 3:

Ch. 3, Supplementary problem 22 (sodium).

Ch. 5, Supplementary problems 8 (epedemic), 12 (neuron), 15 (microtubules).

If anything on this assignment looks like it might be a typo, do not hesitate to email me about it.